QPSK Balanced Space Time Trellis Codes

T.M.H. Ngo, P. Viland, G. Zaharia, J-F. Hélard

Abstract: All the best known space-time trellis codes (STTCs) belong to the Balanced STTCs class. The first method to design balanced STTCs is proposed in this letter. The exhaustive code search of optimal codes can then be reduced to this class. Finally, the proposed method is used to construct new QPSK codes for 3 and 4 transmit antennas which outperform the best previously published codes of the same complexity.

Introduction: The concept of space-time trellis codes (STTCs) was introduced by Tarokh *et al.* in [1]. Several design criteria to search the optimal STTCs have been proposed in [1, 2]. All the published codes which achieve the best performance have the same property : they use the points of the constellation with the same probability if the data are generated by a binary memoryless source with equally probable symbols. Therefore, we call these codes "Balanced STTC" (B-STTC).

To overcome the difficulties of exhaustive codes search in terms of computer resources for a high number of transmit antennas and a high number of states of the encoder, the first method to design this new B-STTCs class is presented in this letter. This method is then used to construct new QPSK B-STTC codes for several transmit antennas which offer better performance than the codes published in [1–3].

System model: In general, for 2^n-PSK modulation, the encoder [4] is composed of one input block of n bits and ν memory blocks of n bits. At each time sample $t \in \mathbb{Z}$, all the bits of a block are replaced by the n bits of the previous block. The i^{th} bit of the j^{th} block x_i^{t-j+1}, with $i = 1 \dots n$ and $j = 1 \dots \nu + 1$, is associated to n_T multiplier coefficients $g_{i,j}^k \in \mathbb{Z}_{2^n}$, $k = 1 \dots n_T$, where n_T is the number of transmit antennas. A ST trellis encoder is thus classically defined by its generator matrix $\mathbf{G}$ of $n_T \times n(\nu + 1)$ coefficients:

$$\mathbf{G} = \begin{bmatrix} g_{1,1}^1 & \cdots & g_{n,1}^1 & \cdots & g_{1,\nu+1}^1 & \cdots & g_{n,\nu+1}^1 \\ \vdots & & & \cdots & & & \vdots \\ g_{1,1}^k & \cdots & g_{n,1}^k & \cdots & g_{1,\nu+1}^k & \cdots & g_{n,\nu+1}^k \\ \vdots & & & \cdots & & & \vdots \\ g_{1,1}^{n_T} & \cdots & g_{n,1}^{n_T} & \cdots & g_{1,\nu+1}^{n_T} & \cdots & g_{n,\nu+1}^{n_T} \end{bmatrix} \tag{1}$$

Thus, the output MIMO symbol $\mathbf{Y} = [y_1 y_2 \dots y_{n_T}]^T \in \mathbb{Z}_{2^n}^{n_T}$ is $\mathbf{Y} = \mathbf{G} \cdot \mathbf{X}$ where $\mathbf{X}$ is a column vector containing each $n(\nu + 1)$ input bits.

Balanced codes: By definition, a STTC is balanced if and only if each generated codeword $\mathbf{Y} = \mathbf{G} \cdot \mathbf{X}$ has the same number of occurrences $n(\mathbf{Y}) = n_0 \geq 1$. In addition, if all the codewords $\mathbf{Y}$ of the constellation are generated, the code $\mathbf{G}$ is a *fully balanced* STTC.

The B-STTC has the following properties:

Theorem 1: The minimum length of a fully balanced STTC is L_{min} =

$dim(\mathbb{Z}_{2^n}^{n_T}) = n.n_T$.

Theorem 2: The addition of one column vector to the coding matrix of a balanced STTC generates a new balanced code.

Theorem 3: Each permutation of the rows/columns of the generator matrix $\mathbf{G}$ of a B-STTC gives the generator matrix $\mathbf{G}'$ of another B-STTC.

Code design: The design of the QPSK fully balanced codes with n_T transmit antennas and $L_{min} = 2n_T$ columns of the generator matrix G has 2 steps:

- First step: generation of all the bases of $\mathbb{Z}_4^{n_T}$.
- Second step: permutation of the column vectors of each obtained base to generate all the fully balanced codes.

The normal subgroup $\mathcal{C}_0 = 2\mathbb{Z}_2^{n_T}$ of the additive group $\mathbb{Z}_4^{n_T}$ allows the partition of $\mathbb{Z}_4^{n_T}$ into 2^{n_T} cosets $\mathcal{C}$ with $\mathcal{C}_p = p + \mathcal{C}_0$, $p \in \mathbb{Z}_2^{n_T}$. A base of $\mathbb{Z}_4^{n_T}$ contains at least one non-null vector in the coset $\mathcal{C}_0$ and at most n_T non-null vectors in $\mathcal{C}_0$. Then, there are n_T types of fully balanced codes according to k, the number of non-null vectors of $\mathcal{C}_0$ used to form a base, with $1 \leq k \leq n_T$.

The algorithm to obtain a type n_T base of $\mathbb{Z}_4^{n_T}$ is as follows:

1. Choose n_T linearly independent vectors of $\mathcal{C}_0$ ($N_B = \frac{1}{n_T!}\prod_{k=0}^{n_T-1}(2^{n_T} - 2^k)$ possibilities) in order to generate $\mathcal{C}_0$.
2. Choose n_T linearly independent vectors $p_1, p_2, \ldots, p_{n_T} \in \mathbb{Z}_2^{n_T}$ (N_B possibilities). Therefore, the cosets $\mathcal{C}_{p_1}, \mathcal{C}_{p_2}, \ldots, \mathcal{C}_{p_{n_T}} \subset \mathbb{Z}_4^{n_T}$ are linearly

independent.

3. Choose the vectors $u_1 \in \mathcal{C}_{p_1}, u_2 \in \mathcal{C}_{p_2}, \ldots, u_{n_T} \in \mathcal{C}_{p_{n_T}}$ (2^{n_T} possibilities for each coset). Because $u_1 + \mathcal{C}_0 = \mathcal{C}_{p_1}$, the whole coset $\mathcal{C}_{p_1}$ is generated. In a similar way, the cosets $\mathcal{C}_{p_2}, \ldots, \mathcal{C}_{p_{n_T}}$ are generated. By doing the combination between the generated cosets, the other cosets are also generated.

Then, there are $N_B^2 \times 2^{n_T^2}$ bases of type n_T of $\mathbb{Z}_4^{n_T}$. Because $dim\ (\mathbb{Z}_4^{n_T}) = 2n_T$, each permutation of $2n_T$ vectors forming a base of $\mathbb{Z}_4^{n_T}$ generates a different balanced code. Therefore, there are $N_C = 2^{n_T^2} \times (2n_T)! \times N_B^2$ fully balanced STTC of type n_T with the minimum length $L_{min} = 2n_T$.

Results and code performance: Table 1 contains all the best 4-state fully balanced codes for $n_T = 2$ transmit antennas. All these codes are of type 2, *i.e.* their generator matrix contains 2 non-null vectors in the coset $\mathcal{C}_0$. All these codes have $\min(\text{rank}(\mathbf{A})) = 2$, $\min(\text{tr}(\mathbf{A})) = 10$ and a minimum product distance $d_p^2 = 24$, and provide the best distance spectrum. In order to confirm the performance of the fully balanced STTC, an exhaustive computer search of all 4-state STTCs has also been carried out. The obtained results confirm that Table 1 contains all the best STTCs. Among them, we found the code proposed by Chen which is in bold in Table 1. There are not other codes with better performance than the codes given in this table.

Table 2 lists the new QPSK STTCs with 3 and 4 transmit antennas. The best known codes found by Chen in [3] are also reported for comparison. The codes are described by the balanced property (fully balanced codes, balanced codes and not balanced codes are noted by FB, B and NB respectively), the number of transmit antennas n_T, the number of states 2^ν, the matrix generator, the minimum trace, and finally a frame error rate (FER) at SNR of 10 dB with the number of receive antennas $n_R = 2$. Some of the new codes have the same minimum trace than Chen's codes, but they achieve a better distance spectrum. In the simulation, each frame consists of 130 symbols from each transmit antenna. A slow Rayleigh fading channel is assumed with variance $\sigma^2 = 0.5$ per dimension. In all cases, we observe that the new balanced codes have a better FER at SNR of 10 dB.

Fig. 1 compares the FER of the new QPSK STTCs given in Table 2 with the FER of Chen's codes for $n_R = 2$. Again, the simulation results show that the new codes outperform Chen's codes of the same complexity.

Conclusion: We have proposed a new class of QPSK Balanced STTCs. These codes generate the points of the MIMO constellation with the same probability. It has been shown that the best STTCs belong to this class. Therefore, the systematic search for good codes can be reduced to this class. Finally, it has been shown that the new proposed codes for 3 and 4 transmit antennas outperform the best previously published codes of the same complexity.

References

[1] V. Tarokh, N. Seshadri and A.R. Calderbank, “Space-time codes for high data rate wireless communication: Performance criterion and code construction”, *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744-765, Mar. 1998.

[2] Z. Chen, J. Yuan and B. Vucetic, “Improved space-time trellis coded modulation scheme on slow fading channels”, *Electron. Lett.*, vol. 37, no. 7, pp 440-441, Mar. 2001.

[3] Z. Chen, B.S. Vucetic, J. Yuan and Ka Leong Lo, “Space-time trellis codes for 4-PSK with three and four transmit antennas in quasi-static flat fading channels”, *IEEE Commun. Lett.*, vol. 6, no. 2, pp. 67-69, Feb. 2002.

[4] T.M.H. Ngo, G. Zaharia, S. Bougeard and J-F. Hélard, “A New Class of Balanced 4-PSK STTC for Two and Three Transmit Antennas”, in *Proc. IEEE SPAWC 2007*, pp. 1-5, June 2007.

Authors' affiliations:

T.M.H Ngo, P. Viland, G. Zaharia and J-F. Hélard (Institute of Electronics and Telecommunications of Rennes (IETR) - INSA - 20 avenue des Buttes de Coesmes, 35043 Rennes, France)

Email : Minh-Hien.Ngothi@ens.insa-rennes.fr

Figure caption:

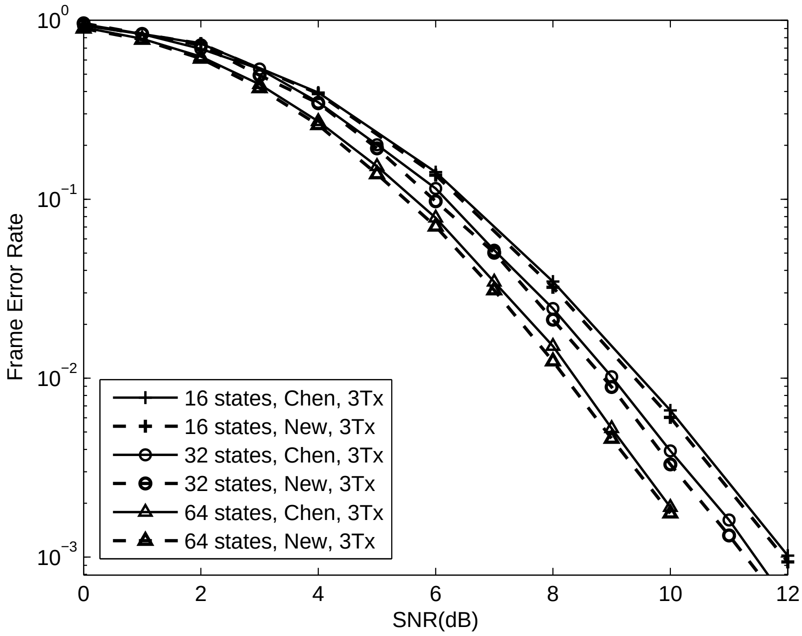
Figure 1 : Performance comparison of QPSK STTCs, $n_R = 2$

Table captions:

Table 1 : 4-state QPSK fully B-STTC with 2 Tx and $\min\left(\mathrm{tr}(\mathbf{A})\right) = 10$

Table 2 : New QPSK STTCs

Figure 1



Frame Error Rate
SNR(dB)
10^0
10^{-1}
10^{-2}
10^{-3}
0
2
4
6
8
10
12
16 states, Chen, 3Tx
16 states, New, 3Tx
32 states, Chen, 3Tx
32 states, New, 3Tx
64 states, Chen, 3Tx
64 states, New, 3Tx

Table 1

$\begin{bmatrix}1 & 2 & 0 & 2\\ 2 & 0 & 2 & 1\end{bmatrix}$	$\begin{bmatrix}1 & 2 & 0 & 2\\ 2 & 0 & 2 & 3\end{bmatrix}$	$\begin{bmatrix}3 & 2 & 0 & 2\\ 2 & 0 & 2 & 1\end{bmatrix}$	$\begin{bmatrix}3 & 2 & 0 & 2\\ 2 & 0 & 2 & 3\end{bmatrix}$
$\begin{bmatrix}2 & 1 & 2 & 0\\ 0 & 2 & 1 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 1 & 2 & 0\\ 0 & 2 & 3 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 3 & 2 & 0\\ 0 & 2 & 1 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 3 & 2 & 0\\ 0 & 2 & 3 & 2\end{bmatrix}$
$\begin{bmatrix}2 & 0 & 2 & 1\\ 1 & 2 & 0 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 0 & 2 & 3\\ 1 & 2 & 0 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 0 & 2 & 1\\ 3 & 2 & 0 & 2\end{bmatrix}$	$\begin{bmatrix}2 & 0 & 2 & 3\\ 3 & 2 & 0 & 2\end{bmatrix}$
$\begin{bmatrix}0 & 2 & 1 & 2\\ 2 & 1 & 2 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 2 & 3 & 2\\ 2 & 1 & 2 & 0\end{bmatrix}$	$\begin{bmatrix}\mathbf{0} & \mathbf{2} & \mathbf{1} & \mathbf{2}\\ \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{0}\end{bmatrix}$	$\begin{bmatrix}0 & 2 & 3 & 2\\ 2 & 3 & 2 & 0\end{bmatrix}$

Table 2

n_T	2^ν	code	matrix	trace	FER SNR=10 dB $n_R = 2$
3	16	Chen B	$\begin{pmatrix} 1&2&1&2&3&2 \\ 2&0&3&2&2&0 \\ 1&2&2&0&1&2 \end{pmatrix}$	24	6.599e-3
		New B	$\begin{pmatrix} 0&2&1&2&2&0 \\ 2&1&2&0&3&2 \\ 2&1&3&2&1&2 \end{pmatrix}$	24	6.033e-3
	32	Chen B	$\begin{pmatrix} 0&2&2&1&1&2&0&2 \\ 2&2&3&2&2&3&0&0 \\ 2&0&3&2&2&1&0&0 \end{pmatrix}$	24	3.923e-3
		New FB	$\begin{pmatrix} 2&3&2&3&0&0&0&2 \\ 2&3&2&1&2&1&0&2 \\ 2&1&0&2&2&3&0&0 \end{pmatrix}$	26	3.296e-3
	64	Chen FB	$\begin{pmatrix} 0&2&3&2&3&0&3&2 \\ 2&2&1&2&3&0&2&0 \\ 2&0&0&2&2&3&1&1 \end{pmatrix}$	28	1.899e-3
		New FB	$\begin{pmatrix} 2&3&2&3&2&1&2&1 \\ 0&2&0&2&2&3&0&2 \\ 2&1&0&2&2&3&2&1 \end{pmatrix}$	32	1.755e-3
4	32	Chen NB	$\begin{pmatrix} 0&2&2&1&1&2&0&2 \\ 2&2&3&2&2&3&0&0 \\ 2&0&3&2&2&1&0&0 \\ 2&1&2&0&1&0&0&2 \end{pmatrix}$	36	1.383e-3
		New B	$\begin{pmatrix} 2&3&2&3&0&0&0&2 \\ 2&3&2&1&2&1&0&2 \\ 2&1&0&2&2&3&0&0 \\ 0&2&2&1&2&1&0&1 \end{pmatrix}$	36	1.305e-3
	64	Chen NB	$\begin{pmatrix} 0&2&3&2&3&0&3&2 \\ 2&2&1&2&3&0&2&0 \\ 2&0&0&2&2&3&1&1 \\ 1&2&2&0&2&1&3&2 \end{pmatrix}$	38	5.758e-4
		New B	$\begin{pmatrix} 0&2&2&1&0&2&2&1 \\ 2&1&2&3&2&3&0&2 \\ 2&3&0&2&2&1&2&3 \\ 2&1&0&2&0&2&2&3 \end{pmatrix}$	40	5.527e-4